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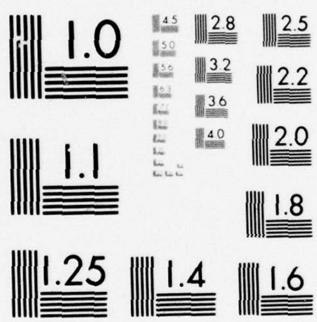
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Constant Radius Magnetic Acceleration of a Strong Nonneutral Proton Ring

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Plasma Physics Division

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The constrained or constant radius magnetic acceleration of a strong, nonneutral proton ring by a modified betatron field is considered. The modified betatron field consists of an azimuthal magnetic field B_θ superimposed on a betatron field B_z in which the 1:2 flux rule is satisfied. An important advantage of the constrained acceleration is that the energy of the nonrelativistic ions increases with the square of the magnetic field. It has been found that the orbits of the gyrating particles are stable when the self-field index $n_s = \{\omega_{pi}^2/[2\gamma_{o\theta}(t)\,\Omega_o^2(t)]\}$ is much greater than unity, provided next page

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20. Abstract (Continued)

B(theta)

cont

that $n_s < (B_\theta/2B_z)^2$ at the orbit. For $B_\theta = o$, the orbits are stable only if $n_s < 1/2$.

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(B(theta)/2Bz) squared

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CONTENTS

CONSTANT RADIUS MAGNETIC ACCELERATION OF A STRONG NONNEUTRAL PROTON RING

I. Introduction

As a result of an intense experimental effort during the last year, pulsed ion beams are presently available[1,2] at a peak power level in excess of 0.2 Terawatts at energies of about 1 MeV. However, several of the fusion related potential applications of ion beams, including target irradiation experiments[3-5] and field reversing ion ring fusion reactors[6] require power levels in excess of 100 Terawatts at energies 10-10,000 MeV. Thus, it is desirable to develop methods of acceleration that would eventually increase the energy of these intense ion beams by as many as four orders of magnitude.

Free or adiabatic magnetic compression of electron rings at the University of Maryland[7] led to a two hundred fold increases of the electron energy. The application of this acceleration method to ion rings has been considered by Fleischmann[8] and Sudan and Ott[9]. In the adiabatic compression the magnetic field varies with time and thus the magnetic moment ($\mu = M_o v_\perp^2/2B$, nonrelativistic) of a gyrating charged particle is an adiabatic invariant. Therefore, the energy of the compressed particles increases linearly with magnetic field. The final value of magnetic field required to accelerate a 1 MeV proton ring of 1 meter initial major radius to about 1 GeV is in excess of 1.4 M Gauss. In addition, extraction and unwinding of the ring after compression into

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a low divergence straight beam is rather difficult. The conservation of the canonical angular momentum P_θ warranties a rapid radial expansion of the beam when a ring made up of ions with $P_\theta \neq o$ is extracted axially through a half-cusp. Radial extraction of the beam is also made difficult because the reduction of the beam major radius with increasing magnetic field excludes the use of deflecting electrodes.

By contrast to the free compression, in the constrained or constant radius magnetic acceleration the magnetic moment is not a constant of the motion and the energy of a nonrelativistic gyrating charge particle increases proportionally to B^2. In addition, radial extraction of the beam can be accomplished using magnetic or electric deflecting fields.

The radius of the ring may be kept constant either by the application of a positive, radial electric field that increases with time or by shaping the external magnetic field in such a way that the betatron flux condition,[11,12] i.e., $< B(t) > = 2B_o(t)$, where $< B(t) >$ is the average and $B_o(t)$ the magnetic field at the orbit of the particle, is satisfied.

In the first approach, the value of the radial electric field required to keep the radius of ions constant depends upon the initial and final value of the axial magnetic field and the initial azimuthal velocity of the ions. The value of the maximum field required for the acceleration of a 3 m radius ring from 2 to 1,000 MeV is about 2.5 MeV/cm. This field is by a factor of 4 higher than the vacuum breakdown field. However, the breakdown in the vacuum gap may be avoided because the externally applied axial field is higher than the critical magnetic field[13] required for magnetic insulation. Details about this method of acceleration will be reported in a future publication.

In this report we present theoretical results on the constant radius acceleration of a strong, nonneutral ion ring in a modified betatron field. Superimposed on the axial magnetic field that satisfies the betatron flux condition is a strong B_θ field. This magnetic field configuration was initially used in the plasma betatron.[14-16] However, the orbit stability analysis in the plasma betatron has been restricted[14] to weak electron beams. As it is shown later on, the B_θ field is necessary for the stability of the orbits when the self fields of the ring are not negligible.

At power levels ~ 10^{12} watts, the constant radius induction ion acceleration competes with several collective ion acceleration schemes. *One such scheme is the ionization front*[17-19]*acceleration process in* which an intense electron beam is used to form a single accelerating electrostatic potential well which traps and accelerates a group of ions. By carefully controlling the ionization rate the ions can be accelerated in a controlled manner to energies many times the electron beam energy. Another class of accelerators rely on the controlled acceleration of ions trapped in a wave which is supported by an intense relativistic electron beam. The Auto Resonant Accelerator[20] (ARA) is of this type. In the ARA the ions are trapped in a beam cyclotron wave which is accelerated on an intense relativistic electron beam by gradually reducing the guiding axial magnetic field. In the Converging Guide Accelerator[21] (CGA) a space charge wave on an intense relativistic electron beam propagating through a cyclindrical waveguide is accelerated by adiabatically decreasing the waveguide radius. In both the ARA and CGA scheme the final phase velocity of the accelerated wave is roughly equal to the electron beam velocity.

II. Induction Accelerations

A low density monoenergetic ion ring in a locally uniform external axial magnetic field will execute gyro-motion around its guiding-center. The particle radius is given by

$$r_o\,(t) = v_{o\theta}(t) / \left[\Omega_o(t)/\gamma_{o\theta}(t)\right] \quad , \tag{1}$$

where $v_{o\theta}$ is the particle velocity perpendicular to the slowly changing local magnetic field B_o, $\Omega_o = |e| B_o / M_o c$ is the nonrelativistic ion cyclotron frequency and $\gamma_{o\theta} = \left[1-(v_{o\theta}/c)^2\right]^{-\frac{1}{2}}$.

The aximuthal electric field at radius r, induced by the changing axial magnetic field is

$$E_\theta(r,t) = \frac{-r}{2c} \; \frac{\partial}{\partial t} \; \langle Bt)\rangle, \tag{2}$$

where $\langle B\rangle$ is the average value of the magnetic field within the radius r. The rate of change of the total energy W of a charged particle of mass M_o and charge $q = |e|$ in the presence of an electric field E_θ is given by

$$\frac{dW}{dt} = |e| \; \vec{v}.\vec{E} \; = |e| v_{o\theta} \; E_\theta, \tag{3}$$

where $W = M_o \; c^2 \gamma_{o\theta}$. Substituting Eqs. (1) and (2) into Eq. (3), it is obtained

$$\frac{\partial}{\partial t} \left\{ \ln \left[\gamma^2{}_{o\theta}(t) - 1\right] \right\} \; \frac{1}{B_o} \; \frac{\partial}{\partial t} \langle B\rangle. \tag{4}$$

Equation (1) is valid only for slowly changing fields, i.e., when

$$\frac{\partial}{\partial t}\left[\ell n\,(B_o)\right] << \Omega_o. \tag{5}$$

Since Eq. (1) has been used in the derivation of Eq. (4), this equation is also valid when the inequality in Eq. (5) is satisfied.

In the case of a uniform, externally magnetic field i.e., $B_o\,(t) = < B(t) >$, Eq. (4) gives

$$\mathcal{E}_i(t) = \frac{\left[\gamma_{o\theta}(o) + 1\right]}{\left[\gamma_{o\theta}(t) + 1\right]}\,\frac{\left[B_o(t)\right]}{\left[B_o(o)\right]}\,\mathcal{E}_i(o), \tag{6}$$

where $\mathcal{E}_i(o) = \left[\gamma_{o\theta}(o) - 1\right] M_o c^2$ is the initial ion energy, $\mathcal{E}_i(t) = \left[\gamma_{o\theta}(t) - 1\right] M_o c^2$ is the final ion energy and $B_o(t)/B_o(o)$ is the ratio of the final to the initial value of magnetic field. -Combining Eqs. (1) and (6), the major radius of the ring $r_o(t)$ is given by

$$r_o(t) = r_o(o)\,\left[B_o(o)/B_o(t)\right]^{\frac{1}{2}}\ . \tag{7}$$

Therefore, in the free or adiabatic acceleration the energy of the non-relativistic ions increases proportionally to the applied magnetic field and the major radius of the ring varies as $B_o^{-\frac{1}{2}}$.

By contrast, when the average magnetic field is twice the local field, i.e., when $< B(t) > = 2B_o(t)$, Eq. (4) gives

$$\mathcal{E}_i(t) = \left[\frac{\gamma_{o\theta}(o) + 1}{\gamma_{o\theta}(t) + 1}\right]\left[\frac{B_o(t)}{B_o(o)}\right]^2\ \mathcal{E}_i(o) \tag{8}$$

Hence, the ion energy in the nonrelativistic case, increases as the square of the magnetic field. This is a significant advantage over the free compression scheme where the energy [Eq. (6)] is proportional to the magnetic field. In addition, Eq. (8) shows that the ratio $c^2 \left[\gamma_{o\theta}^2(t) - 1\right] / \Omega_o^2(t)$ remains constant during compression. However, this ratio is according to Eq. (1) equal to the square of the ring radius. Therefore, when the betatron flux condition $B_o(t) = \langle B(t) \rangle / 2$ is satisfied the radius of the ring remains fixed, although the magnetic field increases with time.

In the conventional betatron accelerators the required spatial variation of the magnetic field is achieved by introducing ferromagnetic material into the core region of the system. In a proton accelerator the presence of ferromagnetic material is not desirable. A ferromagnetic core in a proton accelerator would present rise time and saturation limitations on the field as well as weight problems. However, a field configuration having the proper radial nonuniformity can be easily be achieved with air coils.

III. Orbit Stability in a Modified Betatron Field

In this section we analyze the stability of the single particle orbit in a modified betatron field. In such a field configuration, an aximuthal time independent magnetic field B_θ is superimposed on a time varying betatron field, such that, at $r = r_o$ the condition $B_o = \frac{1}{2} \langle B \rangle$ is satisfied. The present analysis is not restricted to weak rings and thus both the self electric and the self magnetic fields are included. For rings with appreciable self fields, stable orbits exist only if the B_θ field is considerably higher than the initial betatron field B_o.

As shown in Fig. 1, the B_θ field is conveniently produced by passing an axial current along the symmetry axis of the system and the betatron field is generated by two concentric air coils having different diameters.

For the analysis, the external fields are taken as

$$B_z(r,t) = B_o(t)(r_o/r)^n \quad ,$$

$$B_r(r,z,t) = -nB_z(r,t)z/r \quad ,$$

$$B_\theta(r) = B_1 \, r_o/r \quad ,$$

and

$$E_\theta(r,t) = -\frac{1}{rc}\int_o^r \frac{\partial B_z(r',t)}{\partial t} r' \, dr' \quad , \qquad (9a\text{-}d)$$

where n is the external field index, $B_\theta(r)$ is the field produced by the axial current I_o, $E_\theta(r,t)$ is the induced electric field and (r_o, z_o) are the coordinates of the average position of the particles forming the ring. Note that Eqs. (9a,b) are the expressions for the local field and n is assumed constant and uniform over the ring.

To approximate the effects of the self fields we replace the ring by a cylinder of radius a and length $2\pi r_o$. The magnitude of the total force due to the self electric and magnetic field a distance ξ from the axis of the cylinder is

$$F_\xi(t) = \frac{4\pi e^2}{2} \frac{n_i(t)}{\gamma_{o\theta}^2(t)} \xi \quad , \qquad (10)$$

where $n_i(t)$ is the ion density in the ring, which is assumed uniform.

The total self fields of the ring can be represented by an equivalent self electric field of the form

$$\vec{E}_{self}(r,z,t) = \frac{4\pi|e|n_i(t)}{2\gamma_{o\theta}^2(t)} \left[(r-r_o)\,\hat{e}_r + (z-z_o)\,\hat{e}_z\right] \quad , \qquad (11)$$

where $\hat{e}_r$ and $\hat{e}_z$ are unit vectors in the r and z directions. Equation (11) is a good approximation for large aspect ratio rings, i.e., when $(r_o/a) >> 1$.

We now perform a perturbation analysis using the external fields of Eqs. (9) and the equivalent self field of Eq. (11). The particle velocity components are written in the form

$$\begin{aligned} v_r(t) &= v_{or} + \delta v_r \quad , \\ v_\theta(t) &= v_{o\theta}(t) + \delta v_\theta \quad , \qquad (12a\text{-}c) \\ \text{and} \quad v_z(t) &= v_{oz} + \delta v_z \quad , \end{aligned}$$

where $v_{or} = v_{oz} = 0$ and the perturbations δv_r, δv_θ and δv_z are much smaller than v_θ. Substituting the above velocity components into the relativistic orbit equations and keeping terms to first order in the small perturbations we obtain

$$\frac{d}{dt}(\gamma_{o\theta}\delta v_r) + \gamma_{o\theta}\left(\frac{\Omega_o}{\gamma_{o\theta}}\right)^2 (1-n-n_s)\,\delta r$$

$$= -\Omega_1\,\delta v_z - \Omega_o\,\gamma_{o\theta}^2\,\delta v_\theta \quad ,$$

$$\frac{d}{dt}(\gamma_{o\theta}^3 \; \delta v_\theta) = \frac{|e|}{M_o}\left(\frac{\partial E_\theta}{\partial r}\right)\Bigg|_{r = r_o} \delta r \quad ,$$

$$\frac{d}{dt}(\gamma_{o\theta} \; \delta v_z) + \gamma_{o\theta}\left(\frac{\Omega_o}{\gamma_{o\theta}}\right)^2 (n-n_s)\delta z$$

$$\simeq \Omega_1 \; \delta v_r \quad , \qquad \text{(13a-c)}$$

where $\gamma_{o\theta} = \left[1-(v_{o\theta}(t)/c)^2\right]^{-\frac{1}{2}}$,

$n_s = \omega_{pi}^2(t) \Big/ \left[2\gamma_{o\theta}(t)\; \Omega_o^2(t)\right]$ is defined as the self field index,

$\omega_{pi} = (4\pi \; e^2 \; n_i(t)/M_o)^{\frac{1}{2}}$,$\Omega_1 = |e| B_1/M_o c)$ and δr, δz are small spatial perturbations about r_o, z_o, i.e., $d\delta r/dt = \delta v_r$. It can be easily shown from Eq. (9d) that $(\partial E_\theta/\partial r)_{r = r_o} = o$ when the betatron field condition is satisfied. Hence from Eq. (13b) we see that the perturbed angular velocity δv_θ vanishes. Equations (13a) and (13c) can now be put into the rather simple form

$$\delta \ddot{r} + \frac{\dot{\gamma}_{o\theta}}{\gamma_{o\theta}} \; \delta \dot{r} + \omega_r^2 \; \delta r = - \frac{\Omega_1}{\gamma_{o\theta}} \; \delta \dot{z} \quad ,$$

(14a,b)

and $$\delta \ddot{z} + \frac{\dot{\gamma}_{o\theta}}{\gamma_{o\theta}} \; \delta \dot{z} + \omega_z^2 \; \delta z = \frac{\Omega_1}{\gamma_{o\theta}} \; \delta \dot{r} \quad ,$$

where a dot represents the operator d/dt, $\omega_r^2 = (\Omega_o/\gamma_{o\theta})^2 \; (1-n-n_s)$ and $\omega_z^2 = (\Omega_o/\gamma_{o\theta})^2 \; (n-n_s)$. The above equations for the small displacements

δr and δz about r_o and z_o are coupled through the azimuthal magnetic field. The quantities $\gamma_{o\theta}$, Ω_o and n_s are assumed to be "weak" functions of time compared to the cyclotron period.

The self field index n_s, can be expressed in terms of the field reversal factor $\zeta = |B_{self}/B_o|$ where B_{self} and B_o is the self and external magnetic field at the inner surface of the ring. That is,

$$n_s = \frac{\omega_{pi}^2}{2\gamma_{o\theta}\Omega_o^2} = \frac{r_o}{a} \frac{\zeta}{\gamma_{o\theta}^2 \beta_{o\theta}^2} , \tag{15}$$

where $\beta_{o\theta} = v_{o\theta}/c$. Although ζ may be small (a few % field reversal) the self field index can be large, $n_s \gg 1$. Since the magnitude of the external field index is typically of the order of unity, $|n| = O(1)$, the radial and axial betatron frequencies are approximately equal, that is,

$$\omega_r^2 = \omega_r^2 \equiv \omega_o^2(t) < o. \tag{16}$$

Using (16), the condition for the orbits to be stable is shown in appendix A to be given by

$$\epsilon^2(t) < 1, \tag{17}$$

where $\epsilon^2(t) = -\omega_o^2/(\Omega_1/2\gamma_{o\theta})^2 > o$. The stability condition in (17) written in terms of the self field index is $n_s < (\Omega_1/\Omega_o)^2/4$.

In the absence of a B_θ field the orbits are stable if $o < n < 1$ and $n_s < \frac{1}{2}$. The total number of ions and current in the ring is given by

$$N_{ion} = 2 \times 10^{16} \left(\frac{a^2}{r_o}\right) \gamma^3_{o\theta} \beta^2_{o\theta} \, n_s \quad ,$$

and (18a,b)

$$I_{ion} = 1.45 \times 10^{7} \left(\frac{a}{r_o}\right)^2 \gamma^3_{o\theta} \beta^{\,3}_{o\theta} \, n_s \quad ,$$

where a and r_o are in centimeters and I_{ion} is in amperes. Since the field reversal index, number of ions and current in the ion ring is proportional to the self field index, it is desirable, for stability reasons, to have (Ω_1/Ω_o) initially as large as possible.

We now proceed to analyze the solution of the linearized coupled orbit equations Eqs. $\left[(14\ a,b)\right]$ when $n_s >> 1$ and $|n| = O(1)$. To facilitate the analysis we further assume that $\epsilon^2 << 1$. The solutions of Eq. (14) under the above assumptions (see appendix A for a general derivation) are

$$\delta r = e^{\int_o^t \delta(t')dt'} \sum_{n=1}^{4} a_n e^{-i\int_o^t \omega_n^{(o)}(t')dt'} \quad ,$$

and (19a,b)

$$\delta z = e^{\int_o^t \delta(t')dt'} \sum_{n=1}^{4} b_n e^{-i\int_o^t \omega_n^{(o)}(t')dt'} \quad ,$$

where

$$\delta(t) = \frac{1}{2}\frac{d}{dt}(\epsilon^2) \quad , \quad (20)$$

a_n, b_n are constants determined by the initial conditions and $\omega_n^{(o)}$ are the roots of the zeroth order dispersion relation

$$\left[(\omega_n^{(o)})^2 - \omega_o^2\right]^2 - \left(\frac{\Omega_1}{\gamma_{o\theta}}\right)^2 \left(\omega_n^{(o)}\right)^2 = o. \tag{21}$$

Since the frequencies $\omega_n^{(o)}$ are all real the radial and axial dimensions of the ring vary as $\exp \int_0^t \delta(t')dt' \approx 1 + \left[\epsilon^2(t) - \epsilon^2(o)\right] /2.$
The cross sectional area of the ring at time t is, therefore, given by

$$A(t) \approx A(o) \;\{1 + \epsilon^2(o)\,[\epsilon^2(t)/\epsilon^2(o) - 1]\} \quad , \tag{22}$$

where A(o) is the initial cross sectional area. Using the definition of $\epsilon^2(t)$ and the fact that the total number of particles in the ring is conserved we find that

$$\epsilon^2(t)/\epsilon^2(o) = \left[A(o)\,\gamma_{o\theta}(o)/A(t)\gamma_{o\theta}(t)\right] . \tag{23}$$

Combining (22) and (23), the cross sectional area as a function of time is given by

$$A(t) = \left\{1 - \epsilon^2(o)\left[1 - \frac{\gamma_{o\theta}(o)}{\gamma_{o\theta}(t)}\right] A(o)\right\} \quad . \tag{24}$$

Thus, the area A(t) decreases as the magnetic field increases and is bounded by the minimum value $A(t)_{min} = \left[1-\epsilon^2(o)\right] A(o)$.
Furthermore the time evolution of the stability factor ϵ^2 is

$$\epsilon^2(t) = \frac{\gamma_{o\theta}(o)}{\gamma_{o\theta}(t)}\left\{1 + \epsilon^2(o)\left[1 - \frac{\gamma_{o\theta}(o)}{\gamma_{o\theta}(t)}\right]\right\} \epsilon^2(o) \tag{25}$$

$$\approx \frac{\gamma_{o\theta}(o)}{\gamma_{o\theta}(t)} \epsilon^2(o) \quad ,$$

and hence decreases with time. Therefore, the stability of the orbits is improved and the mirror cross section of the ring is reduced during compression.

IV. Discussion and Conclusions

The constrained or constant radius magnetic acceleration of a non-neutral ion ring has two important advantages over the free or adiabatic acceleration. First, the energy of the nonrelativistic ions increases with the square of the magnetic field and second, the extraction of the ions out of the compression system appears to be simpler than in the free compression.

A magnetic field configuration that is suitable for constant radius acceleration of an ion ring is the betatron field, that satisfied the 1:2 flux rule, i.e., the average magnetic field is twice the magnetic field at the orbit of the gyrating charged particle. However, the orbits of the particles in a betatron field become unstable when the self fields of the ring are not negligible. Specifically, the self field index $n_s \left[= \omega_{pi}^2/2\gamma_{o\theta}(t)\Omega_o^2(t)\right]$ must be less than $\frac{1}{2}$ in order for the orbits to be stable. This imposses very stringent limitations on the maximum current of the ring as may be seen from Eq. (18). For 1 MeV protons forming a ring with an aspect ratio $(r_o/a) = 10$ and $n_s = \frac{1}{2}$, this equation states that the maximum current of the ring cannot exceed 10 A. This difficulty is avoided by superimposing an azimuthal magnetic field on the betatron field (modified betatron).

The B_θ field improves considerably the stability of the orbits and permits $n_s >> 1$, provided that $n_s < (\Omega_1/2\Omega_o)^2$, where Ω_1 is the cyclotron frequency corresponding to the B_θ field and Ω_o the cyclotron frequency corresponding to the axial magnetic field at the orbit of the particle.

In addition, the stability of the orbits is improved with the compression because the self field index $n_s \propto \gamma_{o\theta}^{-3}(t)\ \beta_{o\theta}^{-2}(t)$ decreases with time faster than the ratio $(\Omega_1/2\Omega_o)^2$. The same conclusion is also drawn from Eq. (25). Since neither the major nor the minor radius of the ring varies with increasing magnetic field [See Eq. (24)] and since the total number of ions in the ring is concered, Eq. (18a) states that $n_s \propto \gamma_{o\theta}^{-3}(t)\beta_{o\theta}^{-2}(t)$. Substituting this expression for n_s in Eq. (15) we obtain that the field reversing factor ζ also decreases with increasing field and is proportional to $\gamma_{o\theta}(t)$.

The present theoretical model is based on the assumption that the radius of a typical gyrating particle is controlled by the external magnetic field [see Eq. (1)]. Therefore, it is required that the field reversal factor $\zeta \ll 1$. Since ζ decreases with time, the validity of the model is improved with compression. For those applications requiring field reversing magnetic field configurations, the field reversal factor can be increased after the acceleration process by adiabatically compressing it to a smaller radius.

When $n_s \gg 1$ or for arbitrary n_s when $n = \frac{1}{2}$, the betatron frequencies ω_r ans ω_z are equal. As a consequence of this fact the minor cross section of the ring retains its original shape, which is assumed to be circular. Thus, the equivalent electric field in Eq. (11) is a good approximation, provided that the aspect ratio $\left(r_o/a\right) \gg 1$. As stated above, for strong rings, i.e., when $n_s \gg 1$, stability of the orbits requires that the azimuthal magnetic field be much greater than the axial field at $t = o$. For this reason in a practical device the proton beam must be injected into the system along B_θ and not along the symmetry axis of the system. It is rather unlikely that polarization fields will introduce difficulties when the injected beam is space charge neutralized because $(B_\theta/B_o) >> 1$.

The preceeding analysis is also based on the assumption that the ion ring is electrostatically and magnetically noneutral. For neutralized rings the effect of the background electrons on the equilibrium as well as on the stability of the ring must be included. The effect of the electrons on the ion ring depends on the electronic invironment of the ring, that is, whether the neutralized ion ring is in a vacuum or submerged in a dense plasma.

When the ion ring is electrostatically neutralized by background electrons occupying the same volume as the ion ring, the induced electric field E_θ, will produce a zeroth order guiding center drift of electrons towards the axis. This inward radial drift of electrons sets up a radial well as a $\underset{\sim}{E}_r \times \underset{\sim}{B}_z$ drift which tends to enhance the ion current. The radial polarization field, set up by the initial $\underset{\sim}{E}_\theta \times \underset{\sim}{B}_z$ drift, acts to pull the ions in radially if $\omega_{pi}/\Omega_o >> 1$. However, the presence of a large aximuthal magnetic field, such as $B_\theta >> B_z(t)$ reduces the radial drift of the electrons by the factor $B_z(t)/B_\theta << 1$. If, on the other hand, the ion ring is submerged in a dense plasma background of density much greater than the ring density, the radial polarization electric field set up by the $\underset{\sim}{E}_\theta \times \underset{\sim}{B}_z$ drift of electrons is shorted out by the plasma electrons. Hence, the ion ring maintains a fixed average radius as the axial magnetic field is increased.

The equilibrium and stability of ion rings in the presence of a space charge neutralizing electron background is under investigation with a particle simulation computer code.

Appendix A

In this appendix a general discussion of the solution to the orbit equations given in Eqs. (14a,b) is presented. Since $\gamma_{o\theta}$, ${\omega_r}^2$ and ${\omega_z}^2$ are assumed to be weak functions of time the solutions to (14a,b) can be written in the form

$$\delta r = a\, e^{-i\int_0^t \omega(t')dt'} \quad , \qquad \text{(A1)}$$

$$\delta z = b\, e^{-i\int_0^t \omega(t')dt'} \quad , \qquad \text{(A2)}$$

where a, b are constant and $\omega(t')$ is in general complex. Substituting (A1) and (A2) into Eqs. (14a,b) and noting that $|\dot{\omega}| << |\omega^2|$ we obtain

$$\left\{(\omega^2 - {\omega_r}^2) + i\omega\left(\frac{\dot{\omega}}{\omega} + \frac{\dot{\gamma}_{o\theta}}{\gamma_{o\theta}}\right)\right\} \delta r = -i\omega \frac{\Omega_1}{\gamma_{o\theta}} \delta z, \quad , \qquad \text{(A3)}$$

and

$$\left\{(\omega^2 - \omega_z^2) + i\omega\left(\frac{\dot{\omega}}{\omega} + \frac{\dot{\gamma}_{o\theta}}{\gamma_{o\theta}}\right)\right\} \delta z = i\omega \frac{\Omega_1}{\gamma_{o\theta}} \delta r \quad , \qquad \text{(A4)}$$

where $\omega_r^2 = (\Omega_o/\gamma_{o\theta})^2\,(1-n-n_s)$ and $\omega_z^2 = (\Omega_o/\gamma_{o\theta})^2\,(n-n_s)$.

Combining (A3) and (A4) and keeping only the lowest order terms in the time derivatives of ω and $\gamma_{o\theta}$ the following dispersion relationship correct to first order in $\dot{\omega}/\omega^2$ and $\dot{\gamma}/\gamma^2$ is obtained

$$(\omega^2 - \omega_r^2)\ (\omega^2 - \omega_z^2) - \omega^2\left(\frac{\Omega_1}{\gamma_{o\theta}}\right)^2$$

$$+i\omega\left[2\omega^2 - \omega_r^2 - \omega_z^2\right]\left(\frac{\dot{\omega}}{\omega} + \frac{\dot{\gamma}_{o\theta}}{\gamma_{o\theta}}\right) = 0. \tag{A5}$$

We now let $\omega = \omega^{(o)} + \omega^{(1)}$, (A6)

where $|\omega^{(1)}| << |\omega^{(o)}|$ is such that it satisfied the following zeroth order time dependent dispension relation

$$\left[(\omega^{(o)})^2 - \omega_r^2\right]\left[(\omega^{(o)})^2 - \omega_z^2\right] - (\omega^{(o)})^2\ (\Omega_1/\gamma_{o\theta})^2 = 0\ . \tag{A7}$$

Substituting A(6) into (A5) and using (A7) we arrive at the following expression for $\omega^{(1)}$

$$\omega^{(1)} = \frac{-i}{2}\ \frac{\left[2(\omega^{(o)})^2 - \nu^2 + 4(\Omega_1/2\gamma_{o\theta})^2\right]\left[\dot{\omega}^{(o)}/\omega^{(o)} + \dot{\gamma}_{o\theta}/\gamma_{o\theta}\right]}{2(\omega^{(o)})^2 - \nu^2}\ , \tag{A8}$$

where $\nu^2 = \omega_r^2 + \omega_z^2 + 4(\Omega_1/2\gamma_{o\theta})^2$ and $\omega^{(o)}$ is given by the solution of (A7). The four solutions of Eq. (A7) are

$$\omega_{1,2}^{(o)} = \pm\left(\nu_2^2 + \sqrt{\nu^4 - 4\omega_r^{\ 2}\ \omega_z^{\ 2}}\right)^{\frac{1}{2}}\ /\ \sqrt{2}\ , \tag{A9}$$

and

$$\omega_{3,4}^{(o)} = \pm \left(\nu^2 - \sqrt{\nu^4 - 4\omega_r^{\ 2}\omega_z^{\ 2}}\right)^{\frac{1}{2}} / \sqrt{2} \ . \tag{A10}$$

The relations in (A8-10) together with (A1) and (A2) formally describe the linear evolution of the perturbed orbits.

Noting the definitions of $\omega_r^{\ 2}$ and $\omega_z^{\ 2}$, we see from Eqs. (A9) and (A10) that stability is achieved if the following inequalities are satisfied

$$n_s < \tfrac{1}{2} \left[1 + (\Omega_1/\Omega_o)^2\right] \quad , \tag{A11}$$

and

$$\left[(1 - 2n_s) + (\Omega_1/\Omega_o)^2\right]^2 > 4\ (n-n_s)\ (1-n-n_s) > o\ . \tag{A12}$$

In the special case where $n_s = \Omega_1 = o$ stability is implied if

$$o < n < 1 \quad . \tag{A13}$$

The inequality in (A13) is the well known stability condition for a single particle in the absence of a B_θ field. If $\Omega_1 = o$ and $n_s \neq o$, the orbits are stable if

$$n_s < \tfrac{1}{2} \quad , \tag{A14}$$

is satisfied together with

$$n(1-n) < \tfrac{1}{4} \tag{A15}$$

However, when $n_s = 0$ and $\Omega_1 \neq 0$ the condition for stability is the same as when $n_s = \Omega_1 = 0$, that is, the external field index must satisfy (A13). Hence, in the absence of self fields the regime of stability is not increased by the introduction of an azimuthal magnetic field. A situation of somewhat more practical importance is when the self field index is much greater than unity, i.e., $n_s >> 1$ and the external field index is of the order of unity, $|n| \simeq O(1)$. For this case the stability condition is given by

$$n_s < \frac{1}{4}\left(\frac{\Omega_1}{\Omega_o}\right)^2 \quad . \tag{A16}$$

When $n_s >> 1$, the radial and axial betatron frequencies are approximately equal, i.e., $\omega_r^2 = \omega_z^2 = \omega_o^2$. The stability condition in (A16) can then be stated in the form $\epsilon^2 < 1$ where $\epsilon^2 = \omega_o^2/(\Omega_1/2\gamma_{o\theta})^2$. To obtain explicit expressions for $\omega^{(0)}$ and $\omega^{(1)}$ from Eqs. (A8) and (A9), we consider the case where $n_s >> 1$, $|n| = O(1)$ and $\epsilon^2 << 1$, which corresponds to a dense, stable ring. The zeroth order frequencies in (A9) and (A10) for this case are

$$\omega_{1,2}^{(0)} = \pm\left(\frac{\Omega_1}{2\gamma_{o\theta}}\right)\left((2 + \epsilon^2/2\right) \quad , \tag{A17}$$

and

$$\omega_{3,4}^{(0)} = \mp\left(\frac{\Omega_1}{2\gamma_{o\theta}}\right)\frac{\epsilon^2}{2} \quad . \tag{A18}$$

Substitution of (A17) and (A18) yields the single first order frequency

$$\omega^{(1)} = \frac{i}{2} \epsilon^2 \left(\frac{\dot{\omega}_o}{\omega_o} + \frac{\dot{\gamma}_{o\theta}}{\gamma_{o\theta}} \right) = \frac{-i}{4} \frac{d}{dt} (\epsilon^2) \ . \tag{A19}$$

From Eqs. (A19) we see that the amplitude of the linear oscillations has the form

$$\exp \left[\left(\epsilon^2(t) - \epsilon^2(o) \right) / 4 \right] \ .$$

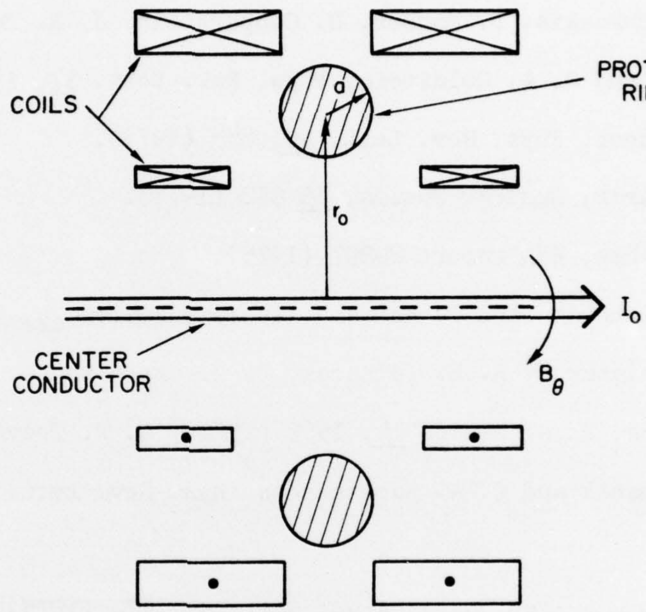

Fig. 1 — Schematic drawing of an apparatus that can be used for the magnetic acceleration of a proton ring. The B_θ field may be produced by the current I_o flowing in the center conductor is independent of time. In the laboratory, the B_θ field may be produced conveniently with toroidal windings. The B_z and B_r fields vary with time and are produced by the two sets of concentric coils. In the proposed scheme, not only the major radius r_o but to a good approximation also the minor radius of the ring remains constant during acceleration.

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